PRIMARY SIX MATHEMATICS
TERM THREE

LENGTH, MASS AND CAPACITY

Conversion of units

1. ***Kilo metres to metres:***

Example: I
Change 7km to metres
1 km = 1000m
7 km = 7 x 1000m
7 km = 7000
Example II
Find the number of metres in 4.7km
1 km = 1000m
4. 7km = 4.7 x 1000m
4.7km = 4700m

2. ***KiloMetres to centimeters.***

Example: I
Convert 2 km to cm.
1 km = 100000cm
2 km = 2 x 100000m
2 km = 200000cm

Example II
Express 3/5 km as cm.
1 km = 100000cm
3/5 km = 3/5 x 100000cm
= 300000cm ÷ 5
= 60000cm

3. ***Metres to centimeters***

Example I:
Convert 4 m to cm.
1 m = 100 cm
4 m = 4 x 100cm
4 m = 400cm

Example II
Express 86m as cm
1m = 100cm
86m = 86 x 100cm
86m = 8600cm

4. ***Metres to km.***

Example I
Write 42000m as km
1000m = 1km.
1m = $\frac{1}{1000}$ km
42000m = $\frac{1}{1000}$ x 42000 km
= 42km

5. ***Centimeters to metres***

Example I
Write 6200cm as metres.
100cm = 1m
1cm = $\frac{1}{100}$ m
6200cm = $\frac{1}{100}$ x 6200 m
= 62 m\.

Example II
Change 530cm to m.
1cm = $\frac{1}{100}$ m
530cm = $\frac{1}{100}$ x 530 m

100

= 5.3 m

<u>6. Square metres to square centimeters</u>

Example I

Write $4m^2$ as cm^2

1m = 100cm

$1m^2 = 100cm^2$

$1m^2 = 100cm \times 100cm$

$1m^2 = 10000cm^2$

$4m^2 = 4 \times 10000cm^2$

$4m^2 = 40000cm^2$

Example II

How many sq cm are there in $20cm^2$?

$1m^2 = 10000cm^2$

$20m^2 = 20 \times 10000cm^2$

7. <u>sq. kilometers to sq. metres</u>

Example I

Express $5km^2$ as m^2

1 km = 1000m

$1km^2 = 1000m^2$

$1km^2 = 1000m \times 1000m$

$1km^2 = 1000000m^2$

$5km^2 = 5 \times 1000000m^2$

$5km^2 = 5000000m^2$

Sq. m to sq. km

Example I

Change $7200000m^2$ to km^2

$1000000m^2 = 1km^2$

$1m^2 = \frac{1}{1000000} km^2$

$7200000m^2 = \frac{1}{1000000} \times 7200000km^2$

$= 7.2km^2$

8. Sq cm to sq m

Example I

Change $190000cm^2$ to m^2

$10000cm^2 = 1m^2$

$1cm^2 = \frac{1}{10000} m^2$

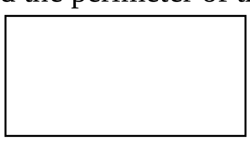
$190000cm^2 = \frac{1}{10000} \times 190000m^2$

$= 19m^2$

PERIMETER OF RECTANGLES

Example:

Find the perimeter of the rectangle below



P = L + W + L + W

P = 50cm + 25cm + 50cm + 25cm

P = 75cm + 75cm

P = 150cm

OR:

P = 2(L +W)

P = 2(50cm + 25cm

P = 2 X 75cm

P = 150cm.

PERIMETER OF SQUARES

Example:

What is the perimeter of the figure below?

7mm

P = S +S +S + S
P = 7mm + 7mm + 7mm + 7mm
P = 28mm

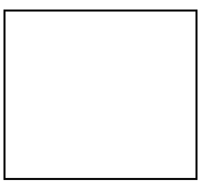
OR:
P = 4 X Side
P = 4 X 7mm
P = 28mm

<u>AREA OF RECTANGLES</u>
Example:
Calculate the area of the figure below:

5cm

6cm
A = L X W
A = 6cm X 5cm
A = 30 cm^2

<u>AREA OF SQUARES</u>
Example:
What is the area of the figure below?



12 cm

A = S X S
A = 12cm X 12cm
A = 144 cm^2
NOTE: area is measured in square units.

<u>APPLICATION OF PERIMETER OF RECTANGLES</u>
Length = <u>perimeter </u>– width

$$\text{Width} = \frac{\text{perimeter}}{2} - \text{length}$$

Example:

1. The perimeter of a rectangle is 18cm and its length is 5cm. Find its width.

$$\text{Width} = \frac{\text{perimeter}}{2} - \text{length}$$

$$\text{Width} = \frac{18\text{cm}}{2} - 5\text{cm}$$

Width = 9cm – 5cm

Width = 4cm

APPLICATION OF PERIMETER OF SQUARES

$$\text{Side} = \frac{\text{perimeter}}{4}$$

Example:

Find the length of a square whose perimeter is 20 dm.

$$\text{Length} = \frac{\text{perimeter}}{4}$$

$$\text{Length} = \frac{20\text{dm}}{4}$$

Length = 5dm

APPLICATION OF AREA OF RECTANGLES

$$\text{Width} = \frac{\text{Area}}{\text{Length}}$$

$$\text{Length} = \frac{\text{Area}}{\text{Width}}$$

Example:

The area of a rectangle is $20cm^2$ and its length is 5cm. Find its width.

$$\text{Width} = \frac{\text{Area}}{\text{Length}}$$

$$\text{Width} = \frac{20\text{cm}^2}{5\text{cm}}$$

Width = 4cm.

APPLICATION OF AREA OF SQUARES

Example:

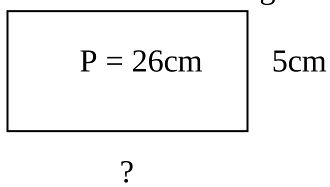
The area of a square is $36cm^2$. Find its side.

FINDING AREA WHEN GIVEN THE PERIMETER

Example:

1. Find the area of the figure below if its perimeter is 26cm and its width is 5cm.

Sketch:



Length = $\frac{\text{perimeter}}{2}$ – width

Length = $\frac{26\text{cm}}{2}$ – 5cm

Length = 13cm – 5cm

Length = 8cm

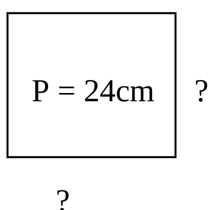
A = L X W

A = 8cm X 5cm

A = 40cm^2

2. The perimeter of a square is 24cm. What is its area?

Sketch:



Side = $\frac{\text{perimeter}}{4}$

Side = $\frac{24\text{cm}}{4}$

Side = 6cm

A = S X S

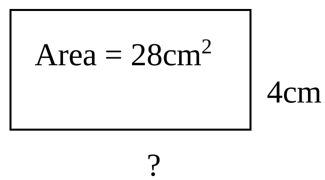
A = 6cm X 6cm

A = 36cm^2

FINDING PERIMETER WHEN GIVEN THE AREA.

1 Find the perimeter of the rectangle whose area is $28cm^2$ and the width is 4cm

Sketch:



$$\text{Length} = \frac{\text{Area}}{\text{Width}}$$

$$\text{Length} = \frac{28cm^2}{4cm}$$

Length = 7cm

P = 2(L + W)

P = 2(7cm + 4cm)

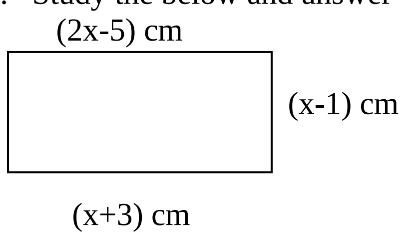
P = 2 X 11cm

P = 22cm

FINDING UNKNOWNS IN SQUARES AND RECTANGLES

Example:

1. Study the below and answer the questions about it:



a. Find the value of x.

2x-5 = x+3

2x-5+5 = x+(3+5)

2x-x = x – x + 8

x = 8

b. What is the length and width of the figure?

Length = x + 3cm

Length = 8 + 3 cm+

Length = 11cm

Width = x – 1cm

Width = 8 – 1cm

Width = 7cm

c. Determine the perimeter of the shape

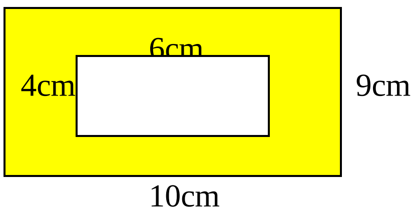
P = 2(L+W)
P = 2(11cm + 7cm)
P = 2X18cm
P = 36cm

d. Calculate the area of the shape.
A = L X W
A = 11cm X 7cm
A = $77cm^2$

FINDING AREA AND PERIMETER OF COMBINED SHAPES

Example 1:

Find the area and perimeter of the shaded region.



Area of outer rectangle:
A = L X W
A = 10cm x 9cm
A = 90 cm^2

Area of inner rectangle:
A = L x W
A = 6cm x 4cm
A = 24 cm^2

Area of shaded part:
A = Outer area – Inner area
A = $90cm^2 - 24cm^2$
A = $66cm^2$

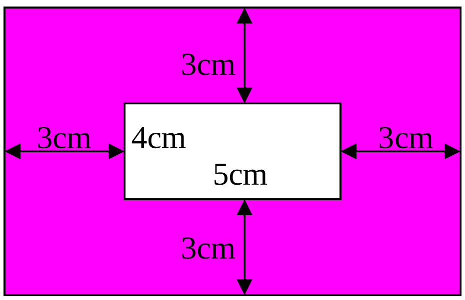
Perimeter of outer figure
P = 2(L + W)
P = 2(10cm + 9cm)
P = 2 x 19cm
P = 38cm

Perimeter of inner figure:
P = 2(L + W)

P = 2(6cm + 4cm)
P = 2 x 10cm
P = 20cm

Perimeter of shaded region:
P = Outer perimeter – Inner perimeter
P = 38cm – 20cm
P = 18cm

Example 2:
Study the figure below and use it to answer the questions that follow:



a. Find the length and width of the outer figure

Length = 5cm +(3cm + 3cm)
Length = 5cm + 6cm
Length = 11cm

Width = 4cm +(3cm + 3cm)
Width = 4cm + 6cm
Width = 10cm

b. Calculate the area of the un-shaded part.
Outer area:
A = L x W
A = 11cm x 10cm
A = $110cm^2$
Inner area:
A = L x W=A = 5cm x 4cm
A = $20cm^2$

Un-shaded area:
A = outer area – inner area.

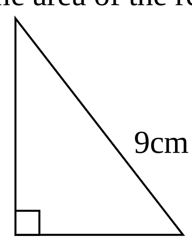
$A = 110\ cm^2 - 20\ cm^2$
$A = 90\ cm^2$

c. What is the perimeter of the un-shaded region?
Perimeter of outer shape:
P = 2(L + W)
P = 2(11cm + 10cm)
P = 2 x 21cm
P= 42cm.
Perimeter of inner shape:
P = 2(L + W)
P 2(5cm + 4cm)
P = 2 x 9cm
P = 18cm

Perimeter of shaded part:
P = outer perimeter – Inner perimeter
P = 42cm – 18cm
P = 24cm.

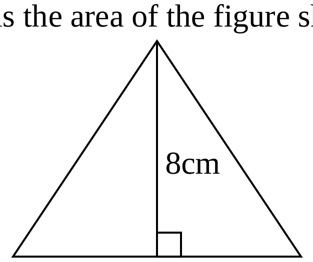
<u>AREA OF TRIANGLES</u>

Example 1:
Find the area of the rectangle below



Area = ½ base x height
A = ½ x 5cm x 6cm
A = 5cm x 3cm
$A = 15cm^2$

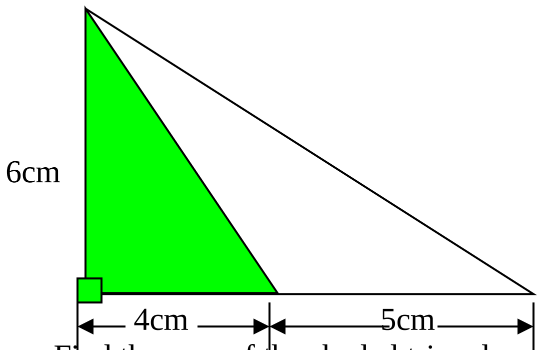
Example 2:
What is the area of the figure shown below?



A = ½ x b x h
A = ½ x 12cm x 8cm
A = 6cm x 8cm
$A = 48\ cm^2$

Example 3:

Study the figure below and use It to answer the questions that follow.



a. Find the area of the shaded triangle.

A = ½ x b x h

A = ½ x 4cm x 6cm

A = 2cm x 6cm

A = $12cm^2$

b. What is the area of the un-shaded figure?

A = ½ x b x h

A = ½ x 5cm x 6cm

A = 5cm x 3cm

A = $15\ cm^2$

c. Calculate the area of the whole figure.

A = ½ x b x h

A = ½ x (4cm + 5cm) x 6cm

A = ½ x 9cm x 6cm

A = 9cm x 3cm

A = $27\ cm^2$

OR:

Whole area = shaded area + un-shaded area.

A = $12\ cm^2 + 15\ cm^2$

A = $27\ cm^2$

FINDING THE BASE OR HEIGHT

Example 1:

Find the base of the triangle whose area is $15\ cm^2$ and the height is 5cm.

½ x base x height = Area

½ b x 5cm = $15\ cm^2$

½ x 5cmb = $15\ cm^2$

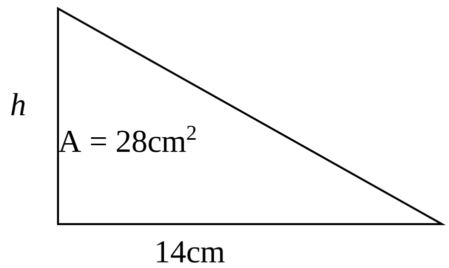
½ (5cmb) x 2 = $15\ cm^2$ x 2

$\frac{5cmb}{5cm} = \frac{30\ cm^2}{5cm}$

b = 6cm

Example 2:

Given the figure below:



Find the value of *h*.

$\frac{1}{2} \times b \times h = \text{Area}$

$\frac{1}{2} \times 14cm \times h = 28cm^2$

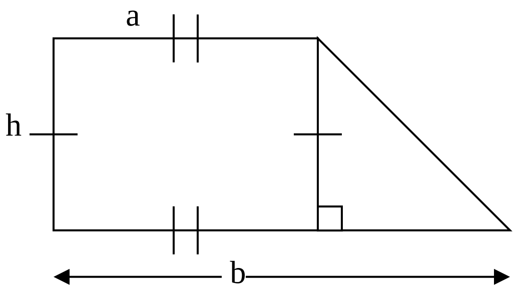
$\frac{1}{2} (14cmh) \times 2 = 28cm^2 \times 2$

$\frac{14cmh}{14cm} = \frac{56cm^2}{14cm}$

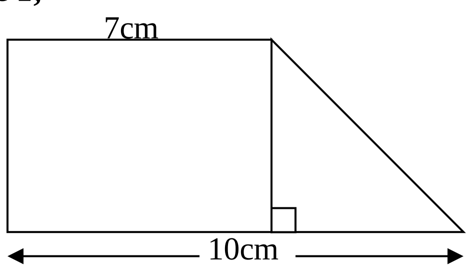
$h = 4cm$

AREA OF TRAPEZIUM

Parts of a trapezium



Example 1;



$A = \frac{1}{2} h (a + b)$

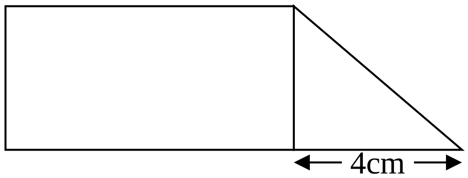
$A = \frac{1}{2} \times 4cm (7cm + 10cm)$

$A = 2cm \times 17cm$

$A = 34 cm^2$

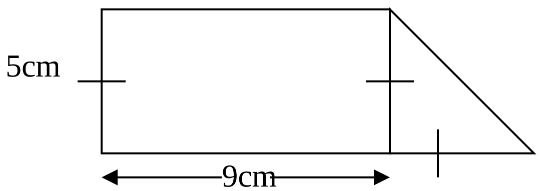
Example 2:

8cm

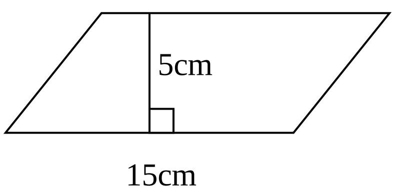


A = ½ h (a + b)
A = ½ x 3cm (8cm + 8cm +4cm
A = ½ x 3cm x 20cm
A = 3cm x 10cm
A = 30cm^2

Example 3:

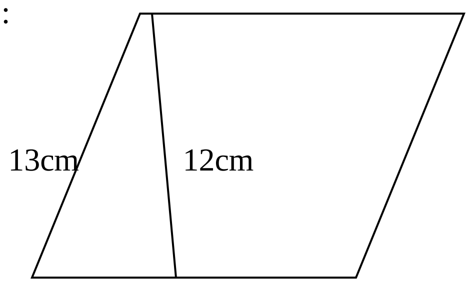


AREA OF A PARALLELOGRAM
Example 1:



Area = base x height
A = b x h
A = 15cm x 5cm
A = 75cm^2

Example 2:



9cm

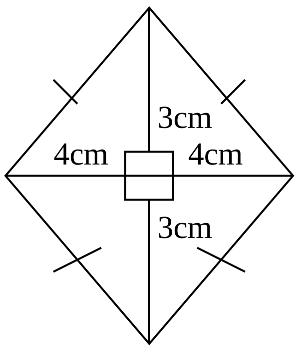
Area = base x height
A = 9cm x 12cm
A = 108cm^2

<u>AREA OF A RHOMBUS AND A KITE</u>

Area = diagonal 1 x diagonal 2

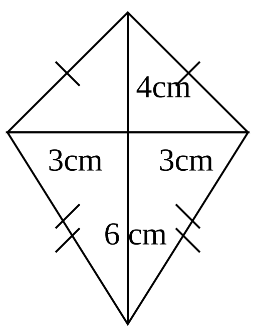
Example 1:

Find the area of the figure below:



A = d1 x d2
A = (3+3) cm + (4+4) cm
A = 6cm x 8cm
A = 48cm^2

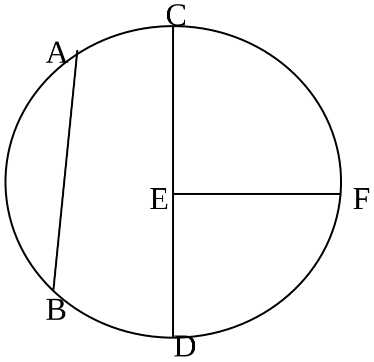
Example 2:



Area = d1 x d2
Area = (3+3) cm x (4+6) cm
Area = 6 cm x 10cm
Area = 60cm^2

CIRCLES

PARTS OF A CIRCLE



AB = Chord
CD = Diameter
EF = Radius
DEF = Sector

Diameter = 2 radii
Radius = ½ diameter
Examples:

1. Find the diameter of a circle whose radius is 3.5cm
   Diameter = 2 x radius
   Diameter = 2 x 3.5cm
   Diameter = 7cm

2. the diameter of a circle is 28cm. Find its radius.
   Radius = ½ diameter
   Radius = ½ x 28cm
   Radius = $\frac{28cm}{2}$
   Radius = 14cm

CIRCUMFERENCE OF A CIRCLE

Circumference is the distance around a circular object.
Circumference = ΠD or 2Πr
Where Π = $\frac{22}{7}$ or 3.14

Example 1:

Find the circumference of a circle whose radius is 14cm
C = 2Πr
C = 2 x $\frac{22}{7}$ x 14cm
C = 44 x 2cm

C = 88cm

Example 2:
The diameter of a circle is 21cm. What is its circumference?
C = ΠD
C = $\frac{22}{7}$ x 21cm
C = 22 x 3cm
C = 66cm

FINDING RADIUS USING CIRCUMFERENCE OF A CIRCLE
Example 1:
Find the radius of a circle whose circumference is 88cm
2Πr = C
2 x $\frac{22}{7}$ x r = 88cm
$\frac{44r}{7}$ x 7 = 88cm x 7
$\frac{44r}{44}$ = $\frac{616cm}{44}$
r = 14cm

FINDING DIAMETER USING CIRCUMFERENCE OF A CIRCLE
Example:
Calculate the diameter of a circle whose circumference is 110cm.
ΠD = C
$\frac{22}{7}$ x D = 110cm
$\frac{22D}{7}$ x 7 = 110cm x 7
$\frac{22D}{22}$ = $\frac{770cm}{22}$
D = 35cm

PERIMETER OF SECTORS OF CIRCLES
Example 1:
Find the perimeter of the shape below.

FINDING AREA OF CIRCLES
Example 1:
Find the area of a circle whose radius is 28cm
A = $Πr^2$

$A = \frac{22}{7} \times 28cm \times 28cm$

$A = 22 \times 28cm \times 4cm$

$A = 2464cm^2$

Example 2:

The diameter of a circle is 14cm.

Calculate its area

$A = \Pi r^2$

$A = \frac{22}{7} \times \frac{14cm}{2} \times \frac{14cm}{2}$

$A = 22 \times 7cm \times 7cm$

$A = 1078cm^2$

<u>FINDING AREA OF A CIRCLE USING CIRCUMFERENCE</u>

Note: first get the radius of the circle and then apply the formula for the area of a circle.

Example 1:

The circumference of a circle is 88cm. Find the area of that circle.

$2\Pi r = C$

$2 \times \frac{22}{7} \times r = 88cm$

$\frac{44r}{7} \times 7 = 88cm \times 7$

$\frac{44r}{44} = \frac{616cm}{44}$

$r = 14cm$

$A = \Pi r^2$

$A = \frac{22}{7} \times 14cm \times 14cm$

$A = 22 \times 14cm \times 2cm$

$A = 616cm^2$

<u>AREA OF DIFFERENT PARTS OF A CIRCLE</u>

Note:

1. Area of a semi-circle = $\frac{1}{2}\Pi r^2$

2. Area of a quadrant = $\frac{1}{4}\Pi r^2$

3. Other sectors, A = angle sector of Πr^2

<u>FINDING RADIUS WHEN GIVEN THE AREA.</u>

Example 1:

The area of a circle is 616cm.

Find its radius.

Πr^2 = area

$\frac{22}{7} \times r^2 = 616cm^2$

$\frac{22r^2}{7} \times 7 = 616cm^2 \times 7$

$\frac{22r^2}{22} = \frac{4312\ cm^2}{22}$

$\sqrt{r^2} = \sqrt{196cm^2}$

$r = 14cm$

FINDING CIRCUMFERENCE OF A CIRCLE GIVEN THE AREA

Find the circumference of a circle whose area is 514 cm^2

Πr^2 = area

$\frac{22}{7} \times r^2 = 514cm^2$

$\frac{22r^2}{7} \times 7 = 514cm^2 \times 7$

$\frac{22r^2}{22} = \frac{1078cm^2}{22}$

$\sqrt{r^2} = \sqrt{49cm^2}$

$r = 7cm$

$C = 2\Pi r$

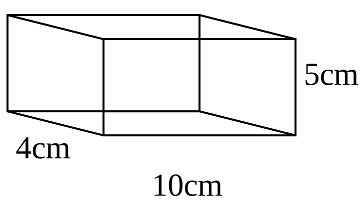
$C = 2 \times \frac{22}{7} \times 7cm$

$C = 44 \times 1cm$

$C = 44cm$

VOLUME OF CUBES AND CUBOIDS

Example 1:

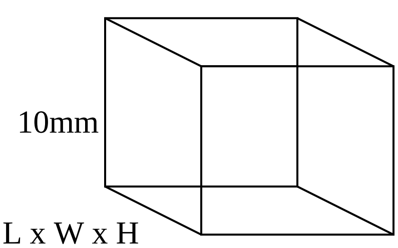


V = L x W x H

V = 10cm x 4cm x 5cm

V = $200cm^3$

Example 2:



V = L x W x H

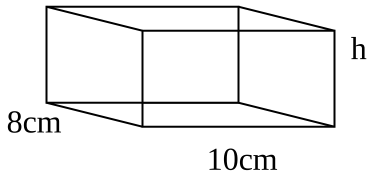
V = 10mm x 10mm x 10mm

V = 1000 mm^3

FINDING THE MISSING SIDE OF A RECTANGULAR PRISM

Example:

The volume of the figure below is $480cm^3$ Find its height.



$l \times w \times h = V$

$10cm \times 8cm \times h = 480cm^3$

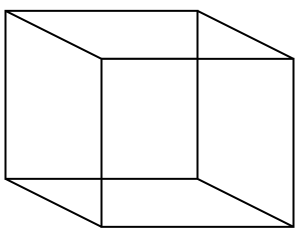
$\frac{80cm^2h}{80cm^2} = \frac{480cm^3}{80cm^2}$

$h = 6cm$

FINDING THE LENGTH OF A CUBE

Example:

The TSA of the figure below is $384cm^2$. Find the length of its side.



$6s^2 = TSA$

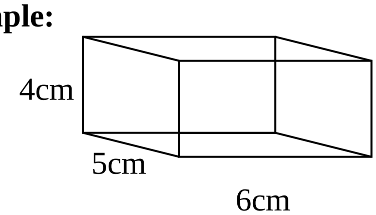
$\frac{6s^2}{6} = \frac{384cm^2}{6}$

$\sqrt{s^2} = \sqrt{64cm^2}$

$s = 8cm$

TOTAL SURFACE AREA OF CUBOIDS

Example:



Length = 6cm

Width = 5cm

Height = 4cm

$TSA = 2(l \times w) + 2(w \times h) + 2(h \times l)$

$TSA = 2(6cm \times 5cm) + 2(5cm \times 4cm) + 2(4cm \times 6cm)$

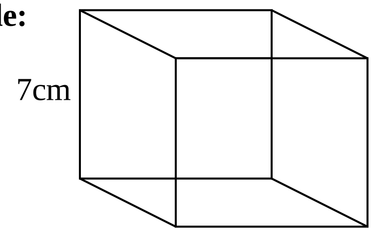
$TSA = 2 \times 30cm^2 + 2 \times 20cm^2 + 2 \times 24cm^2$

$TSA = 60cm^2 + 40cm^2 + 48cm^2$

$TSA = 148cm^2$

TOTAL SURFACE AREA OF CUBES

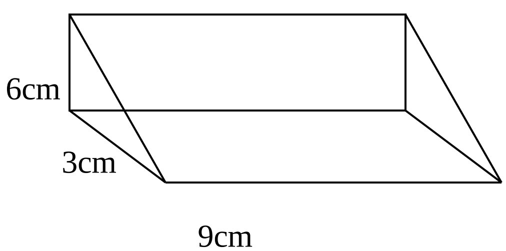
Example:



TSA = $6s^2$
TSA = 6 x 7cm x 7cm
TSA = 6 x $49cm^2$
TSA = $294cm^2$

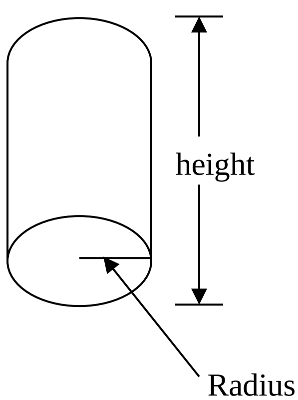
VOLUME OF A TRIANGULAR PRISM

Example:



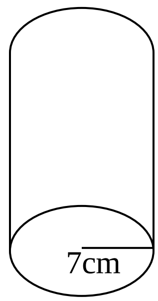
V = area of a triangle x length of the figure
V = ½ x b x h x l
V = ½ x3cm x 6cm x 9cm
V = $81cm^3$

VOLUME OF CYLINDERS



Volume = Area of cross section x height
$V = \Pi r^2 x\ h$

Example:
Find the volume of the figure below:



$V = \Pi r^2 x\ h$
$V = \frac{22}{7} \times 7cm \times 7cm \times 10cm$
$v = 22 \times 70cm^3$
$V = 1540cm^3$

FINDING HEIGHT OF A CYLINDER
Example:

1. The volume of a cylinder is $1584cm^3$ and its base area is $264cm^2$. Find its height.

$\Pi r^2 x\ h = V$
$\frac{264cm^2 h}{264cm^2} = \frac{1584cm^3}{264cm^2}$
$h = 6cm$

2. Calculate the height of a cylinder whose volume is $6776cm^3$ and radius is 14cm.

$\Pi r^2\ x\ h = V$
$\frac{22}{7} \times 14cm \times 14cm \times h = 6776cm^3$
$\frac{616cm^2 h}{616cm^2} = \frac{6776cm^3}{616cm^2}$
$h = 11cm$

CAPACITY

1. How many 2-litre containers can be used to fill a 20-litre container?

20 l ÷ 2l(2litre containers)
10 2-litre containers.

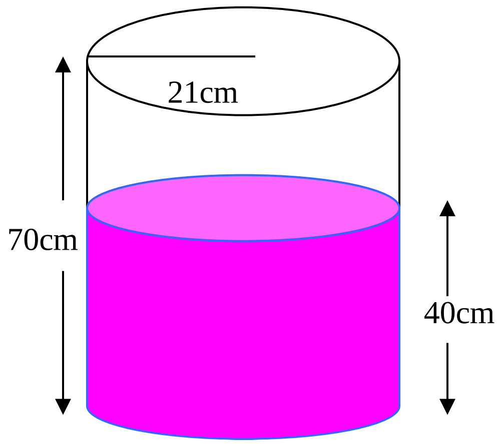
2. Find the number of quarter litre bottles that will be required to fill a container of 50 litres.

50 l ÷ ¼ l containers
50 x 4/1 containers
200-quarter litre containers.

CAPACITY IN VOLUME AND LITRES

Example:

Study the figure below and answer the questions that follow



a. Calculate the number of litres of water in the tank.

$V = \Pi r^2 x\ h$

$V = \frac{22}{7} \times 21cm \times 21cm \times 40cm.$

$V = 55440cm^3$

$C = \frac{volume}{1000cm^3}$

$C = \frac{55440cm^3}{1000cm^3}$

$C = 55.44litres$

b. How much more water is required to fill the tank?

$V = \Pi r^2 x\ h$

$V = \frac{22}{7} \times 21cm \times 21cm \times 30cm.$

$V = 41580cm^3$

$C = \frac{volume}{1000cm^3}$

$C = \frac{41580cm^3}{1000cm^3}$

$C = 41.58litres$

c. Find the capacity of the whole tank.

$V = \Pi r^2 x\ h$

$V = \frac{22}{7} \times 21cm \times 21cm \times 70cm.$

$V = 97020cm^3$

$C = \frac{volume}{1000cm^3}$

$C = \frac{97020cm^3}{1000cm^3}$

$C = 97.02litres$

GEOMETRY

CONSTRUCTING A HEXAGON IN A CIRCLE

Example:

Construct a regular hexagon in a circle of radius 2cm.

1. Draw a circle of the given radius (2cm)
2. Mark point "0" at any point of the circumference of the circle.
3. Using the same radius of the circle, mark off other points 1, 2, 3, 4 and 5 along the circumference of the same circle.
4. Join points 0 to 1, 1 to 2, 2 to 3, 3 to 4, 4 to 5 and 5 to 0 using a ruler and a sharp pencil.

CONSTRUCTING A SQUARE IN A CIRCLE

Example:

Construct a square in a circle of radius 2cm.

1. Construct a square of the given radius (2cm) and draw a diameter AC.
2. Using a pair of compasses, construct another diameter BC that is perpendicular to AC.
3. Using a pencil and a ruler, join the adjacent points ABCD to form a square.

CONSTRUCTING A REGULAR PENTAGON

Example:

Construct a regular pentagon in a circle of radius 2cm.

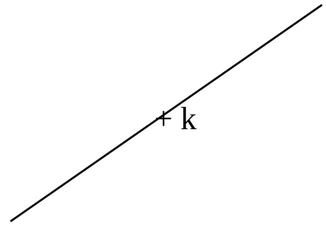
Note: The center angle of a pentagon is 72^0

1. Using a ruler and a protractor, draw an angle of 72^0 at point o.
2. Using o as the center, measure and mark off 2cm along the adjacent arms of the angle and draw a circle .
3. Join the two arcs and measure the length of the line joining the two arcs.
4. Using that as the length of the figure, measure and mark off other points along the circumference.
5. Join all the adjacent points to complete the figure.

CONSTRUCTING PERPENDICULAR LINES

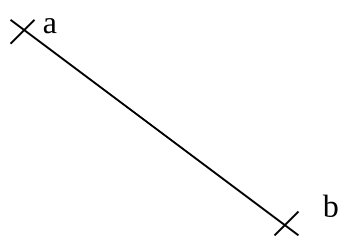
Example: 1.

Construct a perpendicular line at point k



1. Using a pair of compasses and a sharp pencil, make an arc at each end of the line using point k as the center and name them a and b.
2. Using point a as the center, and a bigger radius, make an arc above the line.
3. Again use point b and the same radius to make another arc to intersect the first above the line at point c.
4. Using a ruler and a sharp pencil, draw a line joining point c to point k.

Example 2: Construct a perpendicular bisector for the line below:

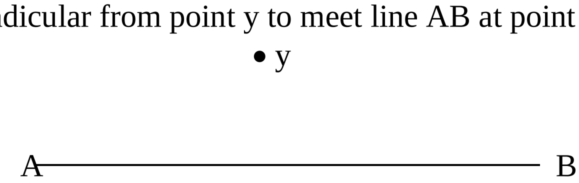


1. Using a pair of compasses and point a as the center, make two arcs below and above the line ab.
2. Using the same radius, change the pair of compasses to point b and use it as the center to construct two other arcs to intersect the first below and above the line ab.
3. Using a ruler and a sharp pencil, draw a line joining the points of intersection of the arcs below and above the line ab through line ab.

CONSTRUCTING PERPENDICULARS FROM POINTS TO LINES

Example:

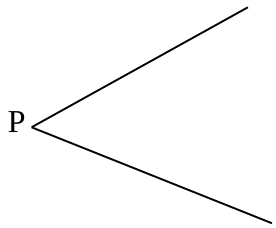
Drop a perpendicular from point y to meet line AB at point x.



1. Using a minimum radius and point y as the center, make two arcs at the line AB, one at each side and name them 1 and 2.
2. Using arc 2 as the cntre and any radius, make an arc at the opposite side of the line.
3. Change to arc 1 and make another arc to bisect the first at point o.
4. Join point y to point o using a ruler and then draw a line from point y to line AB.

5. Name the point of intersection of line AB and the perpendicular line point x.

BISECTING ANGLES
Example:
Bisect the angle below

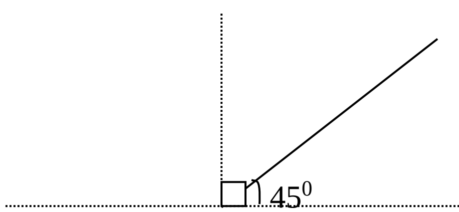


1. Using a minimum radius and point P as the center, make an arc at each of the adjacent arms of the angle and name them 1 and 2.
2. Using the same radius and point 1 as the center, make another arc between the arms of the angle.
3. Turn the position and use point 2 as the center to make another arc between the arms to intersect the first at point r.
4. Using a ruler and a sharp pencil, draw a line to join point r to point P.

Note: 1. The line joining the two points should be prolonged can point r.
2. Bisecting means dividing into two equal parts.

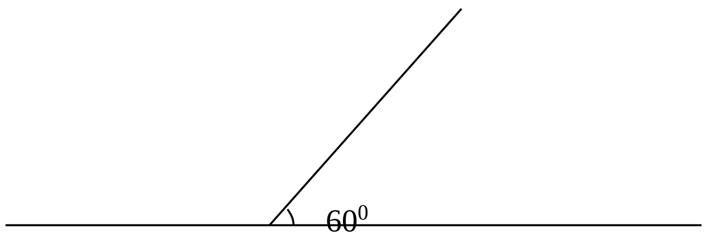
CONSTRUCTING ANGLES
1. 45^0
1. Construct a perpendicular line at a certain point using a dotted line.
2. Bisect one side of the perpendicular.

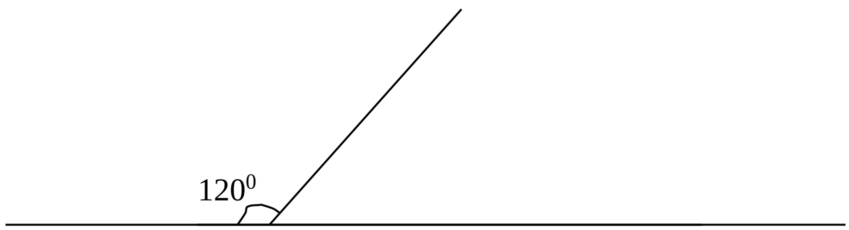
Note: An angle of 45^0 is a half that of 90^0



60^0

1. Draw a line and mark point o along it.
2. Using a pair of compasses and a sharp pencil, measure a minimum radius and use it to make an arc along the line to form point 1 and another arc in the space and name it 2 using point o as the centre.
3. Using the same radius and point 1 as the centre, make another arc to intersect arc 2 at point p.
4. Using a ruler and a sharp pencil, join point o to point p using a solid line.
5. Mark the smaller angle as the angle of 60^0



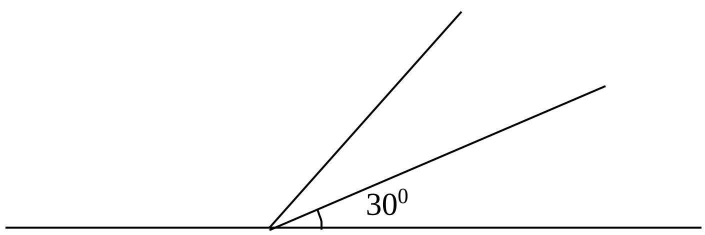


Note: 1. The line joining the two points can go through point p.

2. The bigger angle that remains after 60^0 has been marked is the angle of 120^0

3. 30^0

Note: An angle of 30^0 is a half that of 60^0. So, to construct an angle of 30^0, construct that of 60^0 and bisect it.



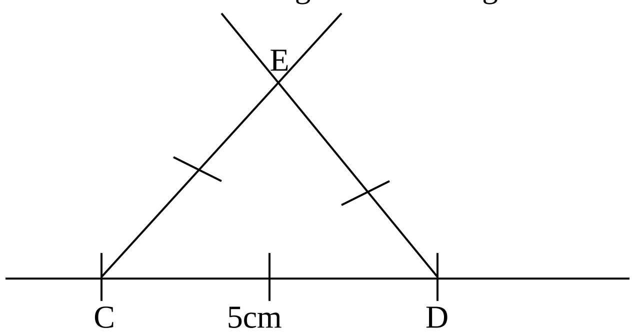
Note: The angle that remains after that of 30^0 has been removed is the angle of 150^0.

CONSTRUCTING AN EQUILATERAL TRIANGLE

Example:

Construct a triangle CDE where CD = DE = EC = 5cm.

1. Draw a line and along it, mark point C.
2. Using a pair of compasses and a sharp pencil, measure 5cm on a ruler and using point C as the center, measure 5cm and mark point D along the line as well as making an arc in the space above the line.
3. Using the same radius and point D as the center, make another arc in the space to intersect the first at point E.
4. Using a ruler and a sharp pencil, join points C and D to point E
5. Mark all the sides using a common sign and indicate the length of only one side.

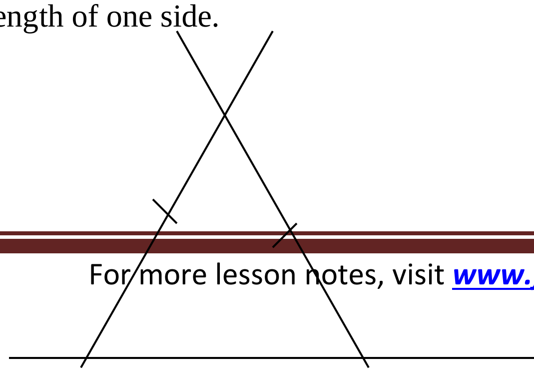


CONSTRUCTING AN ISOSCELES TRIANGLE

Example: Construct triangle MTN where MT = 6cm and TN = NM = 5cm

1. Draw a line and mark points M and T 6cm apart.
2. Measure 5cm on a ruler using a pair of compasses.
3. Using the 5cm and point M and T as the centers respectively, make arcs in the space above the line such that they meet at point N.
4. Use a ruler and a sharp pencil to join points M and T to point N.

5. Indicate the length of MT and mark lines TN and MN using a common mark and indicate the length of one side.



N

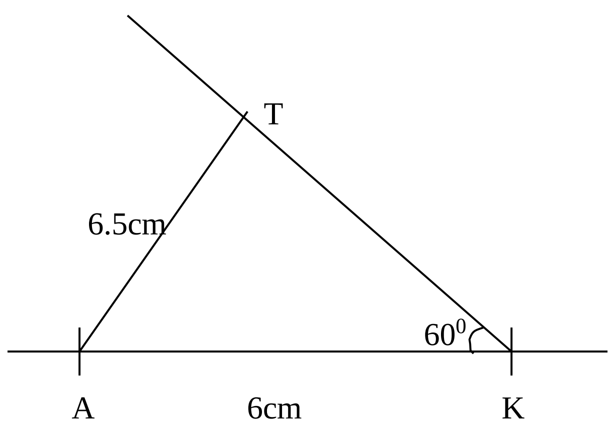
5cm

M 6cm T

CONSTRUCTING A SCALENE TRIANGLE

Example: Construct a triangle TAK when AK = 7cm, AKT = 60^0 and AT = 6.5cm.

1. Draw a line and along it, mark points A and K 6cm apart.
2. At K, construct an angle of 60^0 facing the direction of point A.
3. Measure 6.5cm on the ruler using a pair of compasses and use pint A as the center to make an arc in the space above line AK to intersect the line of 600 at point T.
4. Using a ruler and a sharp pencil, join point T to A.
5. Indicate the length of line AT



CONSTRUCTING PARALLEL LINES.

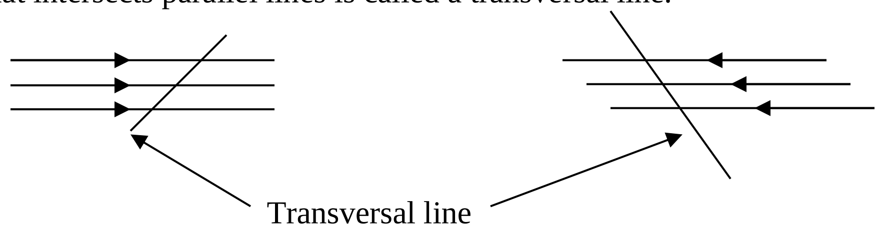
Example:

Construct line CD parallel to line AB.

1. Draw line AB and mark point o along it.
2. At point o, construct a perpendicular line and mark point k along it.
3. At k, construct another line perpendicular to ok.
4. Name the line perpendicular to ok as CD, which is parallel to AB

ANGLE PROPERTIES OF PARALLEL LINES

A line that intersects parallel lines is called a transversal line.



When a transversal line intersects a pair of parallel lines, there are several angles that are formed.

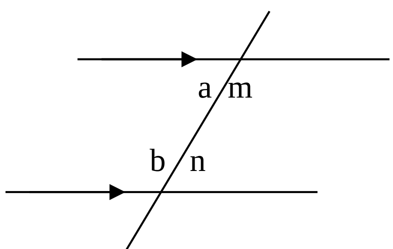
CO-INTERIOR AND CO-EXTERIOR ANGLES

Note:

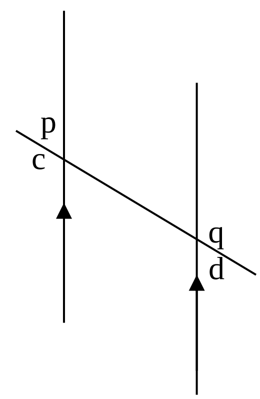
Co-interior and CO-exterior angles add up to 180^0.

Example:

Study the figure below:



$\left.\begin{array}{l}\angle a + \angle b = 180^0 \\ \angle m + \angle n = 180^0\end{array}\right\}$ co-interior angles.

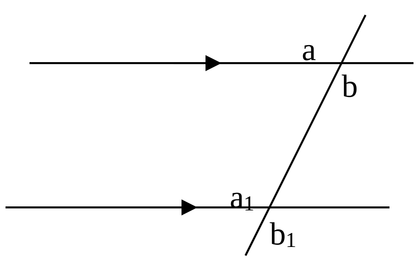


$\left.\begin{array}{l}\angle p + \angle q = 1800 \\ \angle c + \angle d = 180^0\end{array}\right\}$ co-exterior angles

CORRESPONDING ANGLES

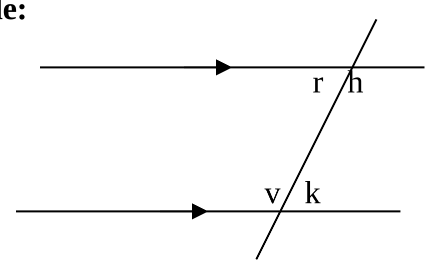
Corresponding angles are equal.

Example:



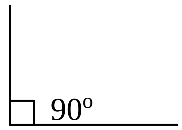
$\left.\begin{array}{l}\angle a = \angle a_1 \\ \angle b = \angle b_1\end{array}\right\}$ Corresponding angles

ALTERNATING ANGLES
Alternating angles are equal.
Example:

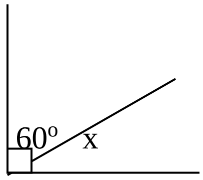


$\left.\begin{array}{l}\angle r = \angle k \\ \angle v = \angle h\end{array}\right\}$ alternating angles.

COMPLEMENTARY ANGLES
Complementary angles add up to 90^o.



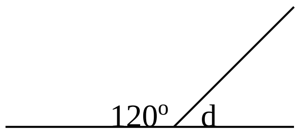
Example:
Find the value of angle x



$x + 60^o = 90^o$
$x + 60^o - 60^o = 90^o - 60^o$
$x + 0 = 90^o - 60^o$
$x = 30^o$

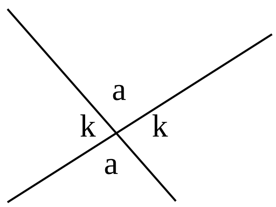
SUPPLEMENTARY ANGLES
Supplementary angles are angles that add up to 180^o.
Supplementary angles are formed on a straight line.

Example:
Find the value of d.



$$d + 120^o = 180^o$$
$$d + 120^o - 120^o = 180^o - 120^o$$
$$d + 0 = 180^o - 120^o$$
$$d = 60^o.$$

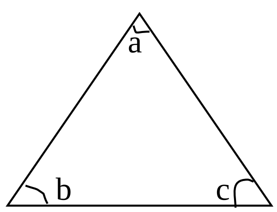
VERTICALLY OPPOSITE ANGLES
Vertically opposite angles are equal.
Example:
Study the figure below:



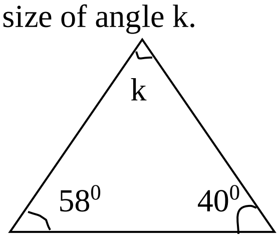
$\left\{ \begin{array}{l} \angle a = \angle a \\ \angle k = \angle k \end{array} \right\}$ vertically opposite angles

$\{\angle a + \angle k = 180^0\}$ supplementary angles

ANGLE PROPERTIES OF A TRIANGLE
The interior angle sum of a triangle is 180^0.

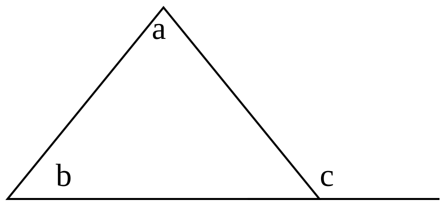


$\{\angle a + \angle b + \angle c = 180^0\}$ interior angle sum of a triangle.
Example:
Find he size of angle k.



$k + 58^0 + 40^0 = 180^0$
$k + \quad 98^0 \quad = 180^0$
$k + 98^0 - 98^0 = 180^0 - 98^0$
$k + 0 \quad = 180^0 - 98^0$
$k \quad = 82^0$

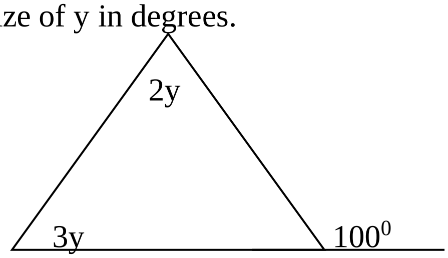
The sum of two adjacent interior angles is equal to the size of the opposite exterior angle.



∠a +∠b =∠ c∠

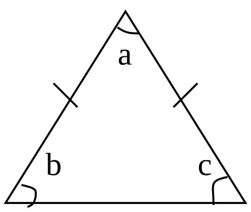
Example:

Find the size of y in degrees.

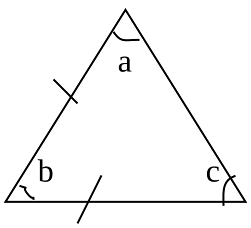


$2y + 3y = 100^0$
$5y = 100^0$
$\frac{5y}{5} = \frac{100^0}{5}$
$y = 20^0$

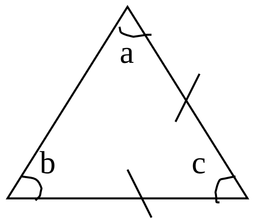
3. An isosceles triangle has two of its adjacent angles equal.



∠b = ∠c

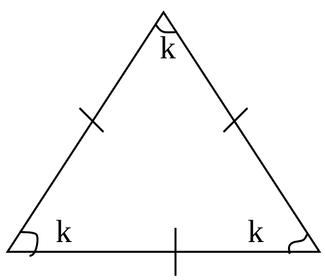


$\angle a = \angle c$



$\angle b = \angle a$

4. A triangle with all sides equal is called an equilateral triangle.
An equilateral triangle has each of its three sides measuring 60^0.
All equilateral triangles are isosceles but not all isosceles triangles are equilateral.



$\angle k + \angle k + \angle k = 180^0$

$3k = 180^0$

$\frac{3k}{3} = \frac{180^0}{3}$

$k = 60^0$.

INTEGERS

Revision

1. Writing the additive integers of others.
2. Arrows on a number line.
3. Showing arrows on a number line.

ADDING A POSITIVE INTEGER TO A POSITIVE INTEGER

Note: A positive integer + a positive integer = a positive integer

Thus: $^+ + ^+ = ^+$

Example:

Add: $^+5 + ^+4$

$^+5 (+) (^+4)$

$^+5 + 4$

$= ^+9$

ADDING A NEGATIVE INTEGER TO A POSITIVE INTEGER

Note: While adding integers of different kinds, the bigger integer dominates.

Thus: $^+ + ^- = ^+/^-$

Example:

1. Add: $^+7 + ^-3$

$^+7 (+) (^-3)$

$^+7 - 3$

$= ^+4$

2. Add: $^{+}3 + ^{-}8$

$^{+}3$ (+) $(^{-}8)$

$^{+}3 – 8$

$= ^{-}5$

ADDING A NEGATIVE INTEGER TO A NEGATIVE INTEGER

Note: Adding a negative integer to a negative integer gives a negative integer.

Thus: $^{-} + ^{-} = ^{-}$

Example: Add: $^{-}5 + ^{-}7$

$^{-}5$ (+) $(^{-}7)$

$^{-}5 – 7$

$= ^{-}12$

SUBTRACTING INTEGERS

SUBTRACTING A POSITIVE FROM A POSITIVE INTEGER

Note: While subtracting a positive integer from a positive integer, the bigger integer after simplifying the signs dominates.

Thus: $^{+} - ^{+} = ^{+}/^{-}$

Example: 1.

Work put: $^{+}4 - ^{+}9$

$^{+}4$ (-) $(^{+}9)$

$^{+}4 – 9$

$= ^{-}5$

2.Simplify: $^{+}8 - ^{+}3$

$^{+}8$ (-) $(^{+}3)$

$^{+}8 – 3$

$= ^{+}5$

SUBTRACTING A POSITIVE INTEGER FROM A NEGATIVE INTEGER

Note: Subtracting a positive integer from a negative integer gives a negative integer.

Thus: $^{-} - ^{+} = ^{-}$

Example: $^{-}5 - ^{+}4$

$^{-}5$ (-) $(^{+}4)$

$^{-}5 – 4$

$= ^{-}9$

SUBTRACTING A NEGATIVE INTEGER FROM A POSITIVE INTEGER

Note: Subtracting a negative integer from a positive integer gives a positive integer.

Thus: $^{+} - ^{-} = ^{+}$

Example:

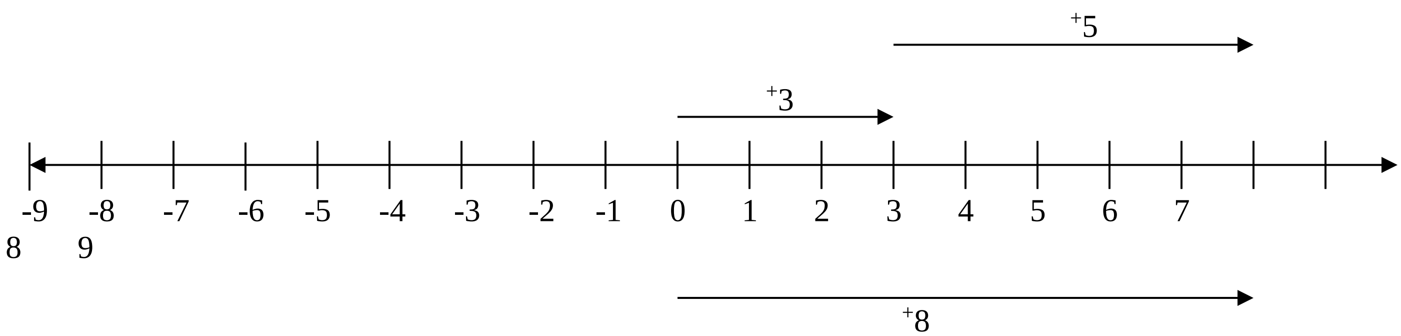
Simplify: $^{+}2 - ^{-}8$

$^{+}2$ (-) ($^{-}8$)
$^{+}2 + 8$
$= {}^{+}10$

SUBTRACTING A NEGATIVE INTEGER FROM A NEGATIVE INTEGER
Note: When subtracting a negative integer from a negative integer, the bigger integer dominates.
Thus: $^{-} - {}^{-} = {}^{-}/{}^{+}$
Example: 1. $^{-}4 - {}^{-}7$
$^{-}4$ (-) ($^{-}7$)
$^{-}4 {}^{+} 7$
$= {}^{+}3$

ADDING INTEGERS ON A NUMBER LINE

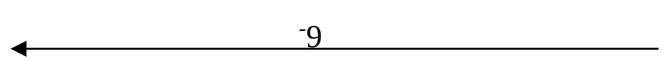
ADDING A POSITIVE INTEGER TO A POSITIVE INTEGER
Example: 1.
Add: $^{+}3 + {}^{+}5$

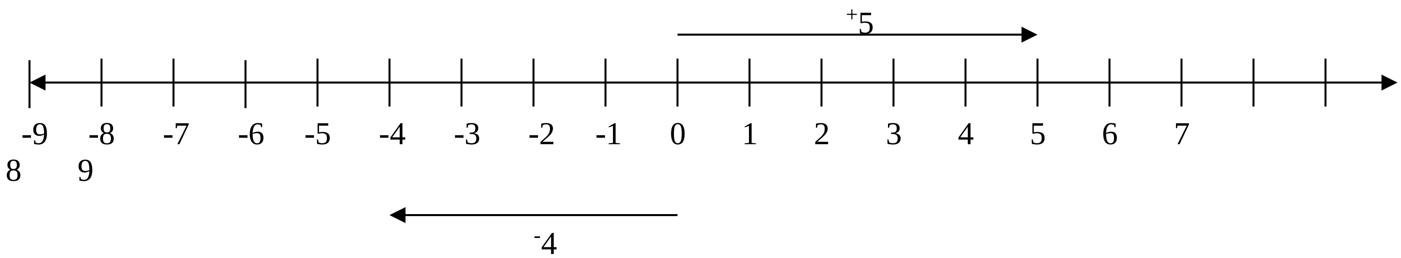


$\therefore {}^{+}3 + {}^{+}5 = {}^{+}8$
Note: Adding a positive integer to a positive integer gives a positive integer.

ADDING A NEGATIVE INTEGER TO A POSITIVE INTEGER

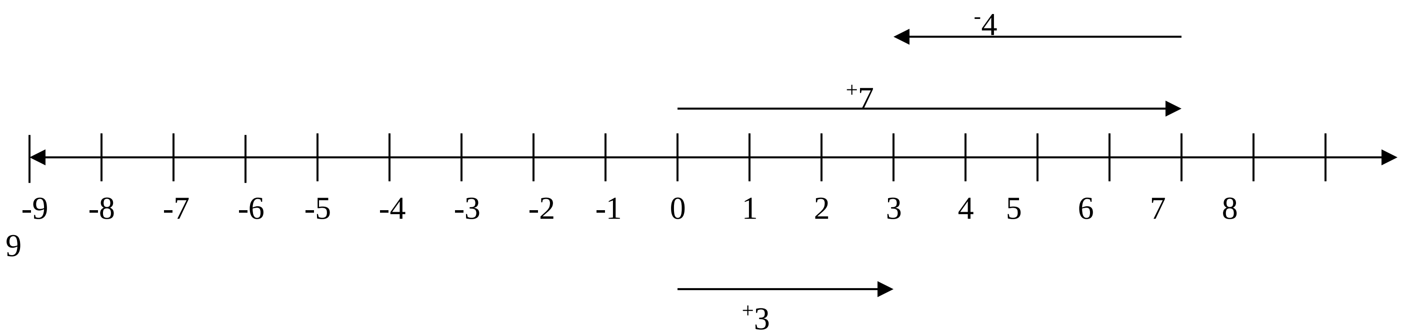
Example 1:
Add: $^{+}5 + {}^{-}9$





$\therefore {}^{+}5 + {}^{-}9 = {}^{-}4$

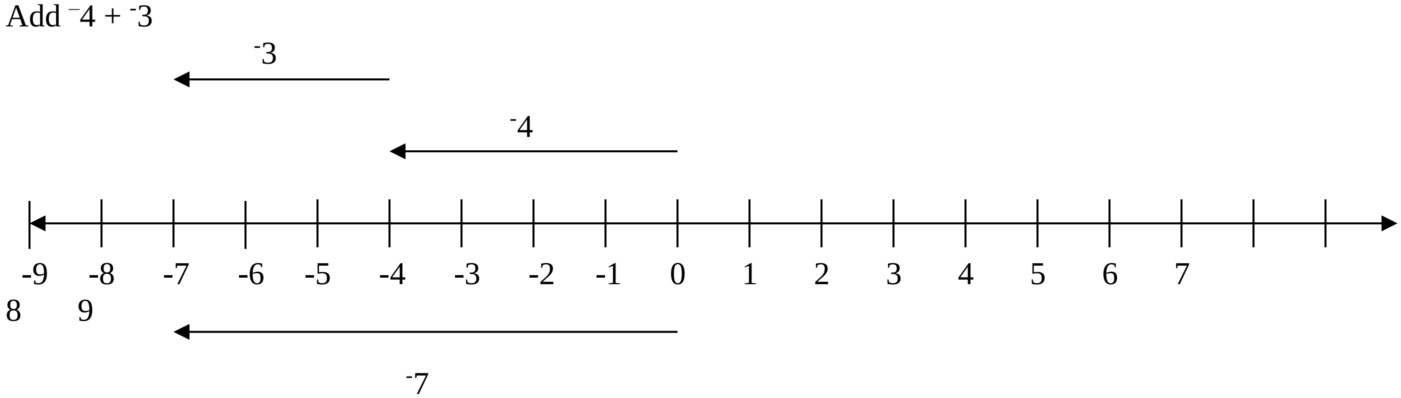
Example 2:
Work out: ${}^{+}7 + {}^{-}4$



$\therefore {}^{+}7 + {}^{-}4 = {}^{+}3$

Note: When adding a negative integer to a positive integer, the bigger integer dominates.

ADDING A NEGATIVE INTEGER TO A NEGATIVE INTEGER
Example:
Add ${}^{-}4 + {}^{-}3$

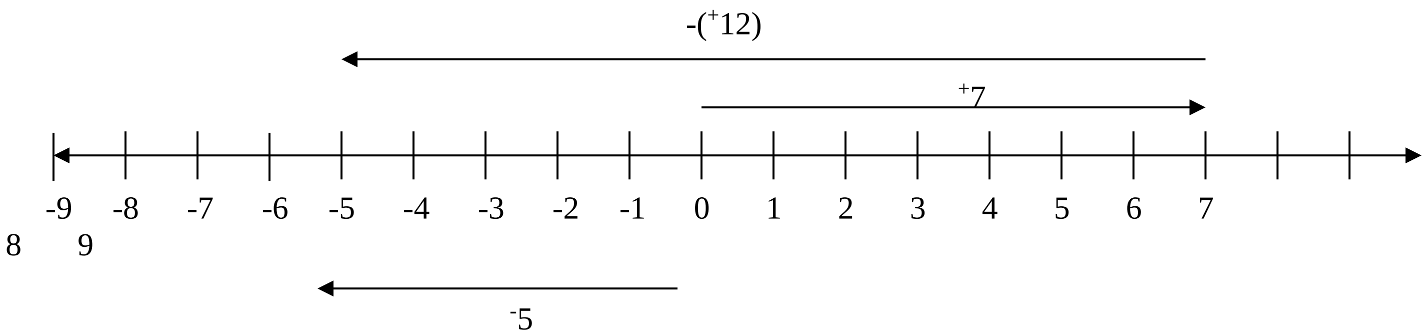


$\therefore {}^{-}4 + {}^{-}3 = {}^{-}7$

Note: Adding a negative integer to a negative integer gives a negative integer.

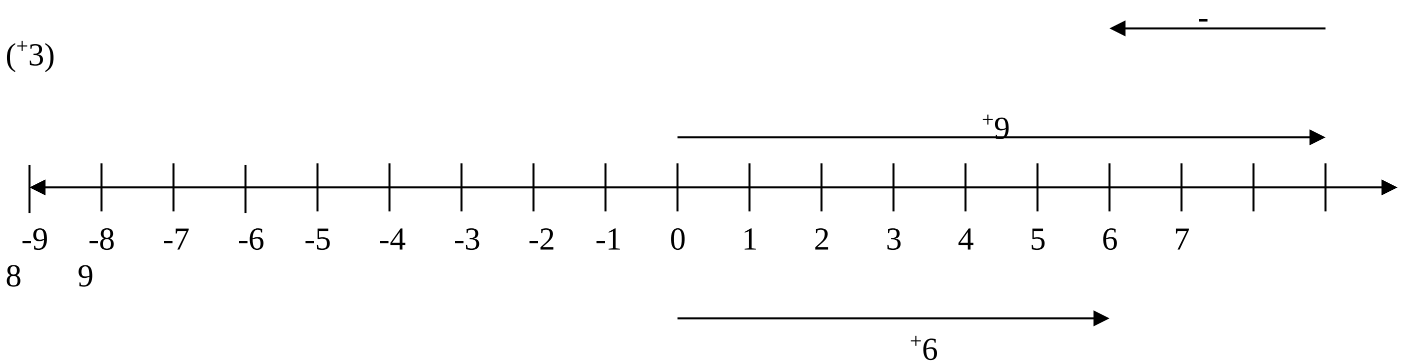
SUBTRACTING INTEGERS ON A NUMBER LINE

SUBTRACTING A POSITIVE INTEGER FROM A POSITIVE INTEGER
Example: 1.
Work out: $^{+}7$ - $^{+}12$



∴ $^{+}7 - ^{+}12 = ^{-}5$

Example 2.
Simplify: $^{+}9$ - $^{+}3$

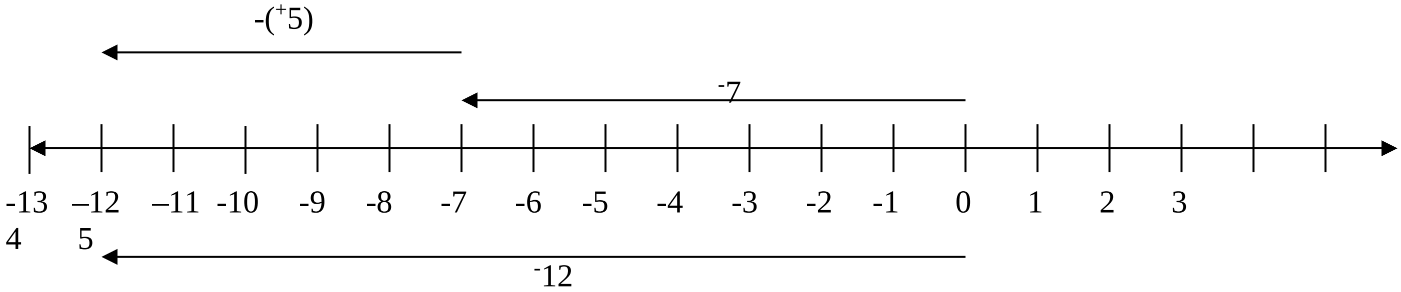


∴ $^{+}9 - ^{+}3 = ^{+}6$

Note: When subtracting a positive integer from a positive integer, if the first integer is bigger, the answer will be positive and where the second integer is bigger, the answer will be negative.

SUBTRACTING A POSITIVE INTEGER FROM A NEGATIVE INTEGER
Example:

Subtract: $^-7 - ^+5$



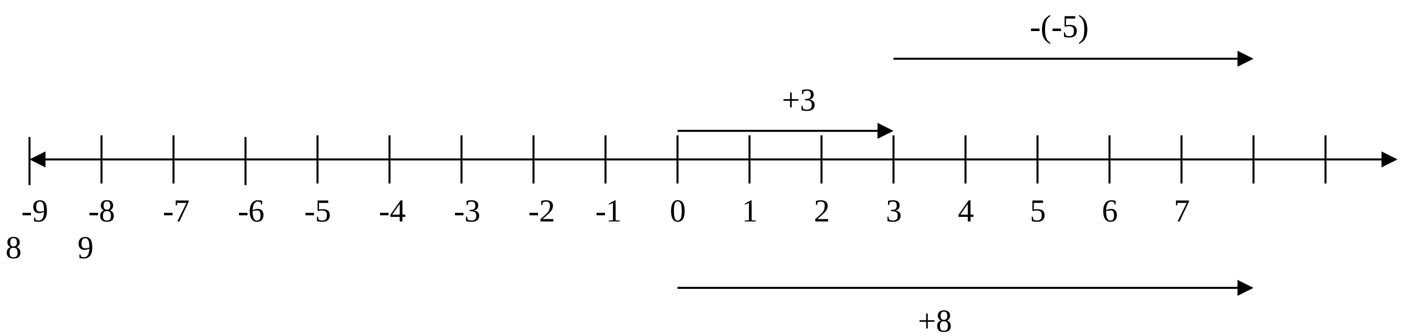
$\therefore\ ^-7 - ^+5 = ^-12$

Note: Subtracting a positive integer from a negative integer gives a negative integer.

SUBTRACTING A NEGATIVE INTEGER FROM A POSITIVE INTEGER

Example:

Simplify: $+3 - -5$



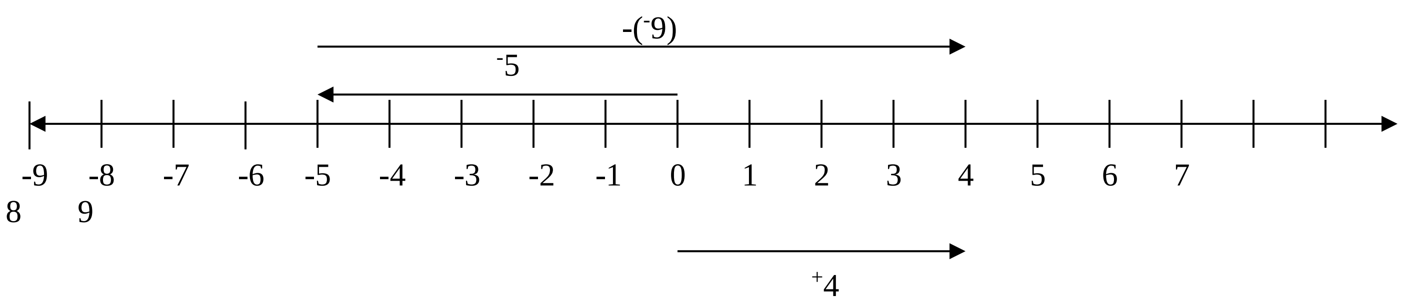
$\therefore\ +3 - -5 = +8$

Note: Subtracting a negative integer from a positive integer gives a positive integer.

SUBTRACTING A NEGATIVE INTEGER FROM A NEGATIVE INTEGER

Example: 1.

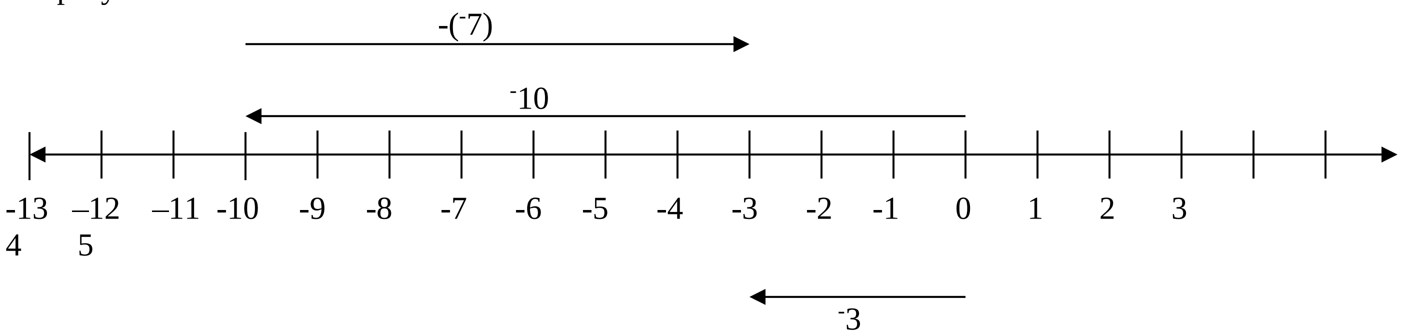
Work out: $^-5 - ^-9$



$\therefore\ ^-5 - ^-9 = ^+4$

Example 2:

Simplify: ${}^{-}10 - {}^{-}7$



$\therefore {}^{-}10 - {}^{-}7 = {}^{-}3$

Note: When subtracting a negative integer from a negative integer, the answer will be negative if the first integer is bigger and where the second integer is bigger, the answer will be positive.

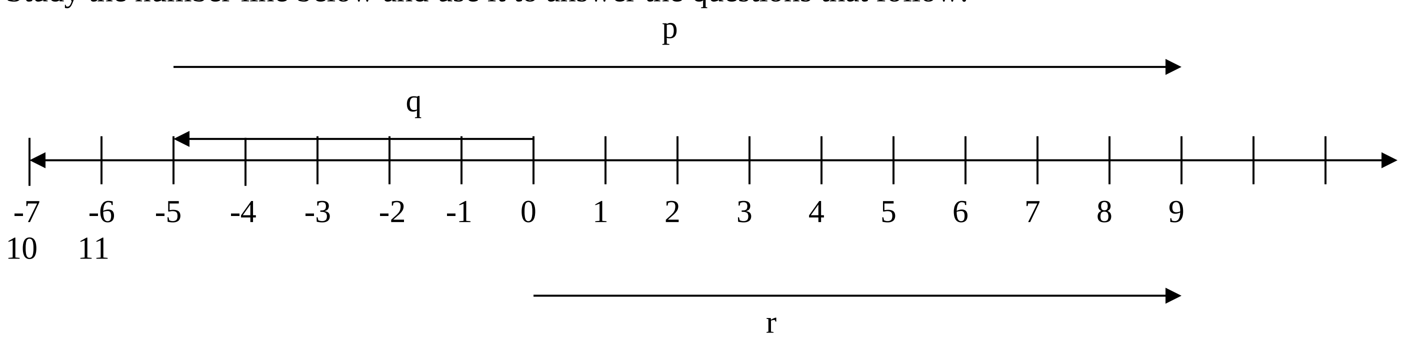
ARROWS ON A NUMBER LINE AND MATHEMATICAL STATEMENTS

Note: 1. Any arrow facing the positive direction represents a positive integer.

2. Any arrow facing the negative direction represents a negative integer

Example:

Study the number line below and use it to answer the questions that follow:



a) Write the integers represented by the arrows:

i. $r = {}^{+}9$

ii. $q = {}^{-}5$

iii. $p = {}^{+}14$

b) Write the mathematical statement shown by the arrows on the number line above.

$q + p = r$

${}^{-}5 + {}^{+}14 = {}^{+}9$

APPLICATION OF INTEGERS

Note: Borrow/lend, debt, fall, loss/lose, drop, decrease and reduce refer to negative while rise, gain and increase refer to positive.

Example:

1. The temperature of a room was 26^0c and it dropped by 45^0c. What is the current temperature?

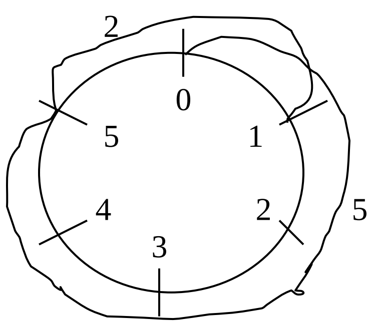
$26^0c - 45^0c$

$= {}^-19^0c$.

FINITE SYSTEM/CLOCK ARITHMETIC

a) Add the following using a clock face.

1. 2 + 5 = ----------- (finite 6)



2 + 5 = 1 (finite 6)

b) Subtract the following using a dial

APPLICATION OF FINITE SYSTEM

Finite 7

Study the table below:

DAY	SUN	MON	TUE	WED	THUR	FRI	SAT
NUMBER	0	1	2	3	4	5	6

Example:

1. Today is Thursday. What day of the week will it be after 58 days from now?

58 ÷ 7 = 8 rem 2
Today + rem
Thur + 2
4 + 2 = 6
6 = Saturday

2. If today is Wednesday, what day of the week will it be 125 days from now?
   125 ÷ 7 = 17 r 6
   Wed + 6
   3 + 6 = 9
   9 ÷ 7 = 1 r 2
   2 = Tuesday
   Therefore, it will be Tuesday

Example 3;
1. Today is Friday. What day of the week was it 44 days ago?
   44 ÷ 7 = 6 r 2
   Day (Friday) – rem
   5 – 2 = 3
   3 = Wednesday.

ALGEBRA

Expressing algebraic expressions in words.
Example
1. k + 3 = sum of k and 3/ add 3 to k
2. 5 – c = subtract c from 5/ difference of 5 and c
3. 3k + 5 = add 5 to the product of 3 and k
4. 3(k + 5) = multiply the sum of 5 and k by 3/ add 5 to k, then multiply the product by 3.

Expressing phrases as algebraic expressions.
Example:
1. Sum of y and 5 = y + 5.
2. Subtract 5 from x = x – 5
3. Triple the difference of x and y = 3(x – y)
4. Square the sum of k and m = $(k + m)^2$

Simplification

Example:
Simplify the following:

1. m + m + m = 3m

2. y + 2x + 3y + x
   y + 3y + 2x + x
   4y + 3x

3. 5m – 2n + 2n – 3m
   5m – 3m + 2n – 2n
   2m + 0
   = 2m

FACTORIZING COMPLETELY

Example:
Factorize completely:

1. mn – n
   n(m – 1)

2. 3ab + ay
   a(3b + y)

3. $15ab^2$ – 5bk
   5b(3ab – k)

Removing brackets
Note: + x + = +
+ x - = -+
- x - = +
- x + = -

Example: Remove the brackets and simplify:

1. (x + 1) + (2x + 2)
   x + 1 + 2x + 2
   x + 2x + 1 + 2
   3x + 3

2. (2y – 1) + (3y – 3)
   2y – 1 + 3y – 3
   2y + 3y – 3 – 1
   5y – 4

3. (3p – 2x) – (p + 2x)

1. (4y – 5) – (y – 5)
   4y – 5 – y + 5
   4y – y – 5 – 5
   3y – 10

3p – 2x – p – 2x
3p – p – 2x – 2x
2p – 2x

More about simplification
Example:
Subtract –2(x + 1) from (3x – 3)
–2(x + 1) from (3x – 3)
(3x – 3) - -2(x + 1)
(3x – 3) + 2(x + 1)
3x – 3 + 2x + 2
3x + 2x + 2 – 3
x – 1

SUBSTITUTION
Example:
a) If a = 4, b = 3, C = $^{-}2$, e = 0 and d = 5: Find the value of:
i) abc
abc = a x b x c
= 4 x 3 x –2
= 12 x –2
= -24

ii) 2ab – ed
2(4 x 3) – (0 x 5)
2 x 12 – 0
= 24 – 0
= 24

iii) $c^b – d^a$
= $^{-}2^3 – 5^4$
= (-2 x-2 x-2) – (5 x 5 x 5 x 5)
= -8 – 225
= -233

b) If a = $\frac{2}{3}$ and b = $\frac{1}{3}$

Find the value of:
a) a + b
$\frac{2}{3} + \frac{1}{3} = \frac{2 + 1}{3} = \frac{3}{3} = 1$

b) $\frac{a}{b}$

$\frac{a}{b} = a \div b$

$= \frac{2}{3} \div \frac{1}{3} = \frac{2}{3} \times \frac{3}{1} = \frac{6}{3} = 2$

SOLVING EQUATIONS

a) by subtraction:

Example 1:

x + 4 = 6

x + 4 = 6

x + 4 – 4 = 6 – 4

x + - = 6 – 4

x = 2

b) by addition:

Example1:

Y – 2 = 7

Y – 2 + 2 = 7 + 2

Y – 0 = 7 – 2

Y = 5

c) by dividing:

Example:

1. 3x = 21

$\frac{3x}{3} = \frac{21}{3}$

x = 7

3. 5m – 8 = 7

5m – 8 + 8 = 7 + 8

5m – 0 = 7 + 8

$\frac{5m}{5} = \frac{15}{5}$

m = 3

d) by multiplying:

Example 1:

$\frac{c}{7} = 6$

$\frac{c}{7} \times 7 = 6 \times 7$

c = 42

WORD PROBLEMS INVOLVING ALGEBRA

Example I

I think of a number. When I add 5 to it, my answer is 12. What is the number?

Soln: Let the number be k.

k + 5 = 12. What is the value of k?

k + 5 = 12

k + 5 – 5 = 12 – 5

k + 0 = 12 – 5

k = 7

so, the number is 7.

Example 2:

The product of two numbers is 130. If one of the numbers is 13, what is the other number?

Soln: Let the other number be n.

n x 13 = 130.

n x 13 = 130

$\frac{13n}{13} = \frac{130}{13}$

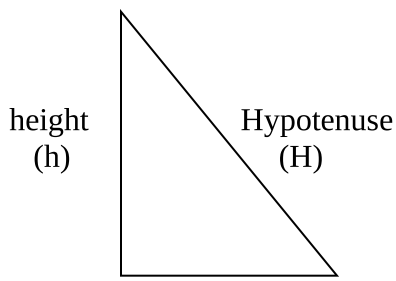
n = 10.

So the other number is 10.

THE PYTHAGORAS THEOREM

The sum of the squares of the short sides of a right-angled triangle is equal to the square of the longest side.

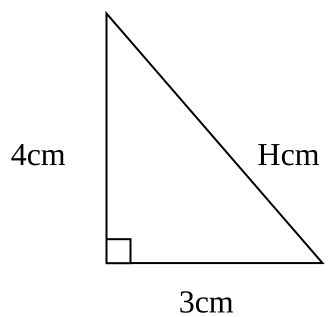
Example:



base (b)

$h^2 + b^2 = H^2$

Considering the figure below:



$$h^2 + b^2 = H^2$$
$$4^2 + 3^2 = H^2$$
$$4 \times 4 + 3 \times 3 = H^2$$
$$16 + 9 = H^2$$
$$25 = H^2$$
$$\sqrt{25} = \sqrt{H^2}$$
$$5 = H$$
$$\therefore H = 5$$